Preliminary work.

This book assumes that you are already familiar with a number of mathematical ideas from your mathematical studies in earlier years.

This section outlines the ideas which are of particular relevance to Unit Two of the *Mathematics Applications* course and for which some familiarity will be assumed, or for which the brief explanation given here may be sufficient to bring your understanding of the concept up to the necessary level.

Read this "preliminary work" section and if anything is not familiar to you, and you don't understand the brief explanation given here, you may need to do some further reading to bring your understanding of those concepts up to an appropriate level for this unit. (If you do understand the work but feel somewhat "rusty" with regards to applying the ideas some of the chapters afford further opportunities for revision, as do some of the questions in the miscellaneous exercises at the end of chapters.)

☞ Chapters in this book will continue some of the topics from this preliminary work by building on the assumed familiarity with the work.

☞ The miscellaneous exercises that feature at the end of each chapter may include questions requiring an understanding of the topics briefly explained here.

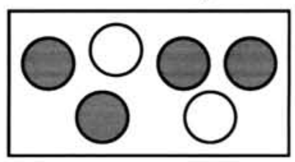
- **Use of number.**

  The understanding and appropriate use of the rule of order, fractions, decimals, percentages, rounding, truncation, square roots and cube roots, numbers expressed with positive integer powers, e.g. 2^3, 5^2, 2^5 etc, expressing numbers in standard form, e.g. $2{\cdot}3 \times 10^4$ (= 23 000), $5{\cdot}43 \times 10^{-7}$ (= 0·000 000 543), also called scientific notation, and familiarity with the symbols $>$, $\geq$, $<$, and $\leq$ is assumed.

- **Ratios.**

  The idea of comparing two or more quantities as a *ratio* should be familiar to you.

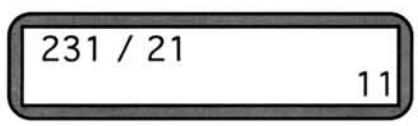
  For example, for the diagram on the right the ratio of unshaded circles to shaded circles is 2 : 4, which simplifies to 1 : 2



  Suppose the ratio of males to females in a school is 17 : 21.

  Knowing there are 231 females in the school we can calculate the number of males:

  Males : females = 17 : 21
  = ? : 231 (×11)

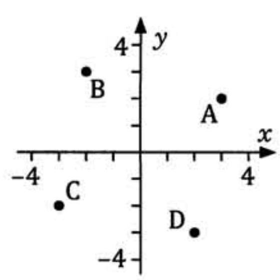


  The number of males = 17 × 11, i.e. 187.

- **Coordinates**

  It is assumed that you are familiar with the idea that points on a graph can be located by stating the coordinates of the point.

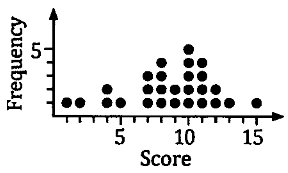
  For example point A has coordinates (3, 2),
  point B has coordinates (−2, 3),
  point C has coordinates (−3, −2),
  point D has coordinates (2, −3).



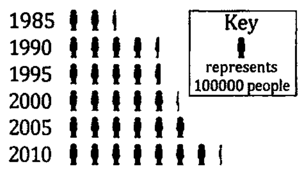
- **Data display**

  It is anticipated that you have already encountered the idea of counting or *tallying* data and organising it into tables, frequency tables and two way classification tables. The following forms of data presentation are also assumed to be familiar.

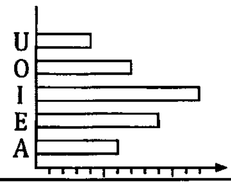
Dot frequency



Pictogram

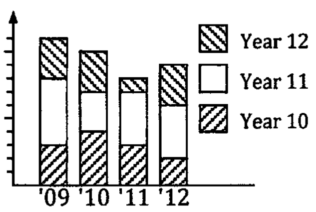


Bar graph (Horizontal bars)

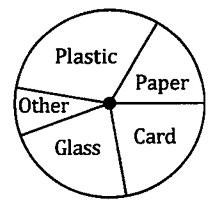


Bar graph (see* below) (vertical bars)

Proportional bar graph



Circle graph

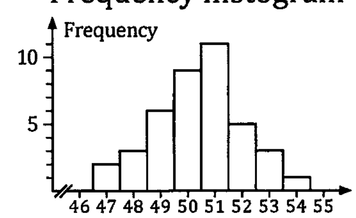


* Some texts call a bar graph with vertical bars a column graph and restrict the name "bar graph" to those with horizontal bars. In this text we will not make that distinction and will refer to both as bar graphs.

Stem and leaf plot

1	14	1 5 5
5 1	15	0 3 8 3
3 2 8	16	3 6 7
0 7	17	8 6
6	18	5

Frequency histogram



- **Data analysis**

  The **mean**, the **median** and the **mode** are all measures used to summarise a set of scores. They are all ways of giving an *average* or *typical* score for the set.
  The **mean** is found by summing the scores and then dividing by the number of scores there are. The mean is the arithmetic average of the scores.

  We use the symbol $\bar{x}$ to represent the mean of a set of scores.
  For example, for the ten numbers: 19, 25, 29, 28, 23, 15, 27, 22, 24, 21

$$\bar{x} = \frac{19 + 25 + 29 + 28 + 23 + 15 + 27 + 22 + 24 + 21}{10}$$

  The mean of the ten numbers is 23·3.
  The mean is a very useful measure of central tendency and is frequently used when analysing data. One disadvantage of the mean is that it can be greatly influenced by extreme scores, called **outliers**. For example if we add an eleventh score of 97 to the above list the mean jumps from 23·3 to 30, i.e. the mean now exceeds all of the original ten scores. Clearly the score of 97 is a long way from the other scores. It is an extreme value and alters the mean significantly.

The **median** is the middle score in an ordered set of scores. If there are an even number of scores we say that the median is the mean of the "middle two" in the ordered set.
For example, to determine the median of the seven numbers

29, 13, 27, 18, 33, 16, 29

we write them in order and choose the middle one:

13, 16, 18, [27], 29, 29, 33

The median of the seven scores is 27.

To determine the median of the eight numbers

29, 13, 27, 18, 33, 16, 29, 13

we write them in order and find the mean of the middle two:

13, 13, 16, [18, 27], 29, 29, 33

The median of the eight scores is 22·5.

The **mode**, or modal score, in a set of scores is the one that appears most frequently. There is of course no guarantee that the mode represents a score that is anywhere near the middle of the set of scores. It can be a useful and informative measure but is not necessarily "central". The mode is used when we want the "most popular" value.
If there are two scores that are equally "most popular" we say the set of scores is **bimodal,** because it has two modes. We do **not** find the mean of the two modes. (Sets of scores with more than two modes could be referred to as multimodal.)
For the ten numbers: 7, 8, 5, 9, 9, 11, 9, 11, 8, 5 the mode is 9.
The set of ten numbers: 5, 8, 5, 9, 9, 11, 9, 11, 8, 5 is bimodal.
The modes are 5 and 9.

Make sure you agree with the given mean, median and mode for the following set of numbers:

3, 3, 1, 4, 3, 0, 5, 5, 5, 3, 1, 4, 5, 4, 2.

The mean of the set of numbers is 3·2, the median is 3 and the set is bimodal with modes of 3 and 5.

As well as wanting to summarise the data using a mean or median or mode we may also be interested to know how widely spread the data is.
One way of indicating this is by stating the **range**, which is simply the difference between the highest score and the lowest score.
For example the set of numbers 7, 8, 5, 9, 9, 11, 9, 11, 8, 5 have a range of 6, obtained by working out (11 - 5).
Whilst the range is easy to calculate it is determined using just two of the scores and does not take any of the other scores into account. For this reason it is of limited use.

- **Formulae.**

  From previous work, probably from *Unit One* of the *Mathematics Applications* course, you should be familiar with the idea of using a formula to determine the value of a variable, or pronumeral, that appears in the formula by itself and on one side of the equals sign, given the values of the variables, or pronumerals appearing on the other side.

  For example:

  Given $A = P + I$ we could determine A, given P and I:

  If $P = 200$ and $I = 15$ it follows that

  $$A = 200 + 15 = 215$$

  Given $C = 2\pi r$ we could determine C, knowing π and given r:

  If $r = 4$, it follows that

  $$C = 2 \times \pi \times 4 = 25{\cdot}133 \text{ (rounded to three decimal places)}$$

  Given $s = ut + \frac{1}{2}at^2$ we could determine s, given u, a and t:

  If $u = 4$, $a = 10$ and $t = 6$ it follows that

  $$s = 4 \times 6 + \frac{1}{2} \times 10 \times 6^2 = 204$$

- **Algebra.**

  You should also be familiar with evaluating **expressions** such as $2x + 3$, $5x - 2$, $5y + 4$, $2(x + 3)$, $3xy + 2z$ etc, given the values of x, y, and z.

  If $x = 2$, $y = 3$ and $z = -5$ then

  $$2x + 3 = 7$$
  $$5x - 2 = 8$$
  $$5y + 4 = 19$$
  $$2(x + 3) = 10$$
  $$3xy + 2z = 8$$

  The idea of **expanding brackets** should also be familiar to you:

  The expression $3(x + 2)$ means "three lots of $(x + 2)$". Think of the bracket as a parcel containing an x and a 2. If we open the three parcels we have three xs and three 2s, i.e. $3x + 6$. We call this *expanding* the brackets.

  Thus $5(x + 4)$ expands to $5x + 20$

  $7(2x + 5)$ expands to $14x + 35$

  $-2(3x - 4)$ expands to $-6x + 8$

  If we are expanding several brackets we may be able to simplify our answer.

  For example $3(2x + 1) + 5(x + 3) = 6x + 3 + 5x + 15 = 11x + 18$

  For example $4(x + 3) - 3(x + 2) = 4x + 12 - 3x - 6 = 1x + 6$ Usually written: $x + 6$.

- **Similar triangles.**
  To know whether two triangles are similar we can:
  - See if the 3 angles of one triangle are equal to the 3 angles of the other triangle.

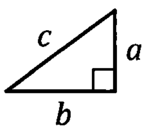
  OR
  - See if the lengths of corresponding sides are in the same ratio.

  OR
  - See if the lengths of two pairs of corresponding sides are in the same ratio and the angles between the sides are equal.

  Once we know that two triangles are similar then we know that corresponding sides are in the same ratio.

- **The Pythagorean theorem.**
  From unit one of this course you should be familiar with the following fact:
  The square of the length of the hypotenuse of a right angled triangle is equal to the sum of the squares of the lengths of the other two sides.

  Thus, for the triangle shown on the right, $c^2 = a^2 + b^2$

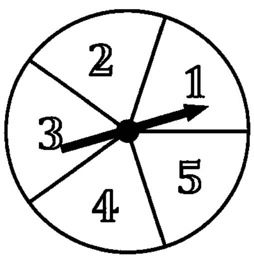
  The theorem of Pythagoras allows us to determine the length of one side of a right triangle, knowing the lengths of the other two sides.

- **Probability.**
  The work of chapter 13 assumes a basic understanding of the idea that the probability of something happening is a measure of the likelihood of it happening and this measure is given as a number between zero (no chance of happening) to 1 (certain to happen).

  If we roll a normal die once the probability of getting a 3 is one sixth.

  We write this as: $P(3) = \frac{1}{6}$.

  For each spin of the spinner shown on the right

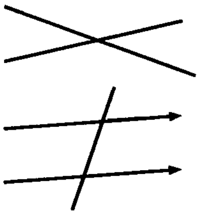
  

  $P(1) = \frac{1}{5}$, $P(2) = \frac{1}{5}$,

  $P(\text{odd number}) = \frac{3}{5}$, $P(\text{even number}) = \frac{2}{5}$,

  $P(\text{number} > 5) = 0$, $P(\text{number} < 6) = 1$.

- **Geometry.**
  It is assumed that you are familiar with the fact that when two straight lines intersect, the vertically opposite angles are equal, and with angle facts relating to alternate angles, corresponding angles and co-interior angles with parallel lines.

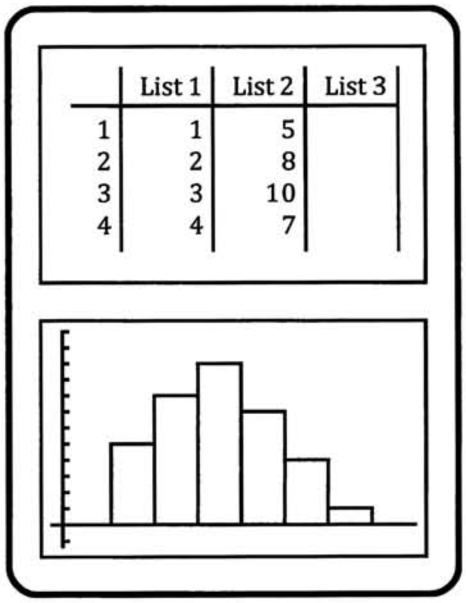
  

- **Use of technology to process and display data.**

  Students following this course may come to this unit with very varied abilities in the use of technology such as graphic calculators and computers. Whatever your initial ability you are encouraged to make use of such technology whenever appropriate.

As the course progresses, do try to:

(a) gain familiarity with entering data into the columns of a calculator with statistical capabilities and showing various statistical information and displays about that data.



and (b) become familiar with entering data into the columns of a spreadsheet on a computer or calculator and of carrying out straightforward operations on those entries such as adding a list of numbers, finding their average and presenting the data as a graph, as shown below.

	A	B	C	D	E	F	G
1		Test 1	Test 2				
2	Alex	9	20				
3	Ben	6	8				
4	Chris	8	17				
5	Diane	7	16				
6	Eric	9	17				
7	Donelle	10	20				
8	Fran	2	7				
9	Gert	4	9				
10	Harri	8	17				
11	Icolyn	9	19				
12							
13	Out of	10	20				
14	Total	72	150				
15	Mean	7.2	15